2016 Smart Grids Conference (SGC), 20-21 Dec. 2016, Graduate University of Advanced Technology, Kerman, Iran

An Optimal Event-triggered Tracking Control for Battery-based Wireless Sensor Networks

Amir Alavi, Mehrnaz Javadipour, Ali .A Afzalian
Faculty of Electrical Engineering
Shahid Beheshti University
Tehran, Iran

Abstract— **Energy consumption in battery-based nodes and network traffic optimization are two main challenges associated with wireless sensor networks and the traditional digital control architecture is not optimized from these perspectives. Thus, using optimal control theory and event-based control concept, we propose a novel architecture to tackle the regulation and tracking problems in wireless sensor networks. The proposed algorithm is realized in MATLAB software as well as a laboratory setup. The result show, the proposed algorithm is able to reduce network traffic and subsequently energy consumption in the nodes, significantly.**

Keywords—*Embedded controller; Wireless sensors networks; Optimal event-based control; Energy saving; Network traffic reduction, Event-Triggered Control (ETC), Event-based regulator, Event-based tracking.*

I. Introduction

Networked Control Systems (NCSs) have been received considerable attention in recent years. NCSs controllers are implemented based on embedded processors and are connected with corresponding sensor and actuator nodes through a communication network. In this structure, not only wiring and maintenance cost will be reduced, but also flexibility for adding/removing and locating remote devices will be increased. NCSs have been reported in various applications, such as power systems [1], aircrafts [2], process control [3], wireless sensor and actuator networks [4] and modern vehicles [5].

Event-based methods on control systems are intended to reduce communication between sensors, controller and the actuators in a control loop [6]-[7]. Exchanging information between the components of the system depends on the triggered event. The events will be generated in order to guarantee the desired control objectives, based on significant changes in the states of the system.

Many research studies have been done on event-triggered control systems. In [8] the problem of scheduling control tasks on embedded processors, to guarantee asymptotic stability of the overall system, is addressed. Authors in [9] proposed event-triggered strategies for control of discrete-time systems and extended the results for self-triggered strategy. Similarly, in [10], discrete-time systems are analyzed where the authors proposed a Lyapunov based triggering mechanism, in which the values of some Lyapunov functions are calculated at each sampling instant and based on the model of system, the value of the Lyapunov function for next sampling instant is predetermined. Therefore, the triggering mechanism can verify whether the Lyapunov function is decreasing or not. References [11] and [12] introduced Periodic Event-Triggered Control (PETC) which deals with discrete-time model of the system. A significant contribution of PETC is that the triggering condition is verified periodically at the sampling times.

While there have been some efforts in past to study analysis and design of event-based control system [13], [14], their systematic design for tasks such as stabilization have been undertaken only recently [8],[15]-[16]. Of these, [17] has significantly influenced the proposed architecture in this paper.

In this paper, we propose a novel architecture for networked controller design considering, event-based communication scheme. Here we focus on linear systems with two general goals: Regulator and Tracking control. Stability analysis for both regulator and tracking control problem, as well as systematic optimal event-based controller design for wireless control systems, are also provided.

II. Event-Based Stabilization of Nonlinear Control Systems

Consider the control system (1) for which a feedback controller $u = k(x)$ has been designed.

$$\dot{x}(t) = f\big(x(t), u(t)\big), \qquad x(t)\epsilon\mathbb{R}^n, u(t)\epsilon\mathbb{R}^m, \quad t\epsilon\mathbb{R}_0^+ \tag{1}$$

Event-based calling of this function leads to computing $u(t_i) = k(x(t_i))$ and updating the actuator values at t_i. We used zero-order hold (ZOH) to implement this function, thus we have:

$$t\epsilon[t_i, t_{i+1}] \Rightarrow u(t) = u(t_i) \tag{2}$$

In traditional digital control systems, the interval between control value updates, $t_{i+1} - t_i = \tau$ is constant. Since in this paper, the control input is updated based on event triggering rule, this constant interval is no more correct. In other words, we would like to determine when it is indeed necessary to execute the control law in order to achieve the desired performance.

Error vector is defined as the difference between the last

978-1-5090-4988-2/16/$31.00 ©2016 IEEE

sampled state vector and its current value of it:

$$e(t) = x(t_i) - x(t) \qquad for\ t\epsilon[t_i, t_{i+1}] \tag{3}$$

Using (3), the closed loop dynamic $\dot{x}(t) = f(x(t), k(x(t_i))$ can be written as:

$$\dot{x}(t) = f(x(t), k(x+e)) \tag{4}$$

Let the control law $u = k(x)$ render the system (1) Input to state stable (ISS) with respect to the continuous feedback error. Under this assumption, there exists a Lyapunov function V for the system that satisfies the following inequality:

$$\dot{V} \leq -\alpha(|x|) + \gamma(|e|) \tag{5}$$

Where α and γ are K_∞ functions. Stability of the closed loop system (4) can be guaranteed if:

$$\gamma(|e|) \leq \sigma\alpha(|x|), \quad \sigma > 0 \tag{6}$$

Therefore, the dynamic of V will be bounded by

$$\dot{V} \leq (\sigma - 1)\alpha(|x|) \tag{7}$$

Taking $\sigma < 1$, V is guaranteed to decrease along plant trajectory.

If $\alpha^{-1}$and γ are Lipschitz function on compact sets, inequality (7) is implied by a simpler inequality

$$b|e| \leq \sigma a|x| \tag{8}$$

where $a\ and\ b$ are Lipschitz constants appropriately chosen according to α^{-1} and γ. Thus, for the stability of the closed loop system, whenever (9) is met the control task should be executed.

$$|e| = \sigma\frac{a}{b}|x| \tag{9}$$

Upon the execution of control tasks we have $x(t_i) = x(t)$, and thus the error becomes zero. Correctly implementation of this condition needs continuous monitoring of the state vector is needed in order to evaluate condition correctly (9). analog circuit or periodic event-triggered control to evaluate (9)[9].

The inter-execution time which implicitly defined by (9) is the time it takes for $|e|/|x|$ to evolve from $0\ to\ \sigma\frac{a}{b}$. We denote this time by :

$$\tau(x(t_i)) := \min\{t > t_i : |e(t, x(t_i))| = \sigma\frac{a}{b}|x(t, x(t_i))|\} \tag{10}$$

In [8] it has been proved that there is a minimum inter-event time for a given event-trigger condition. Thus the event triggered control system can be implemented effectively on hardware such as microcontrollers and FPGAs. In the next section we are going to develop a more specific design for linear systems. The minimum inter-event time is also calculated for the linear systems.

III. Event-Based Regulation of Linear Systems

In order to reach a mature design framework, we focus on linear systems, in this section.

Consider a linear system described by (11):

$$\frac{d}{dt}x_p = A_p x_p + B_p u, \qquad x_p\epsilon\mathbb{R}^{n_p}, u\epsilon\mathbb{R}^{n_u} \tag{11}$$

which has been stabilized by (12)

$$u = Kx_p \tag{12}$$

thus the closed-loop system would be

$$\frac{d}{dt}x_p = A_p x_p + B_p K x_p \tag{13}$$

If the system is asymptotically stable, we have eigenvalues of $A_p + B_p K$, negative. We are going to implement this control law in an embedded controller in wireless control system.

Consider following Lyapunov function candidate:

$$V(x_p(t)) = {x_p}^T P x_p \tag{14}$$

with it's time derivative

$$\frac{d}{dt}V(x_p(t)) = \frac{\partial V}{\partial x_p}(A_p + B_p K)x_p = -{x_p}^T Q x_p \tag{15}$$

for origin to an asymptotically stable equilibrium point, V has to be positive definite and its time derivative must be negative definite. Moreover, the rate at which V decreases is specified by the Q. If we are willing to tolerate a slower rate of decrease, we would require the solution of an event-triggered implementation to satisfy a weaker inequality.

$$\frac{d}{dt}V(x_p(t)) \leq -\sigma {x_p}^T Q x_p \qquad \sigma\epsilon[0,1] \tag{16}$$

σ is a real number between [0,1] specifying the coefficient of the decrease. Event is triggered when (16) is violated. So this equation is the event condition and when violated, the state is sent through wireless medium to the controller. The control law will be executed accordingly.

$$u(t) = u(t_k) \qquad \forall t\epsilon[t_k, t_{k+1}], \qquad k\epsilon\mathbb{N} \tag{17}$$

error $e(t)$ is defined by:

$$e(t) = x_p(t_k) - x_p(t) \qquad \forall t\epsilon[t_k, t_{k+1}], \qquad k\epsilon\mathbb{N}. \tag{18}$$

Using this error we express the dynamics of the closed loop system during the interval $[t_k, t_{k+1}[$ by:

$$\begin{aligned} \frac{d}{dt}x_p(t) &= A_p x_p(t) + B_p u \\ &= A_p x_p(t) + B_p K x_p(t_k) \\ &+ B_p K\left(x_p(t) - e(t)\right) \\ &= A_p x_p(t) + B_p K x_p(t) + B_p K e(t) \end{aligned} \tag{19}$$

The time derivative of V is then

$$\begin{aligned} \frac{d}{dt}V(x_p(t)) &= \frac{\partial V}{\partial x_p}(A_p + B_p K)x_p(t) + \frac{\partial V}{\partial x_p}B_p K e(t) \\ &= -{x_p}^T(t) Q x_p(t) \\ &+ 2{x_p}^T(t) P B_p K e(t) \end{aligned} \tag{20}$$

Applying inequality (16), we will reach to event trigger condition:

$$[x_p{}^T(t) \quad e^T(t)]\begin{bmatrix}(\sigma-1)Q & PB_pK \\ K^TB_p^TP & 0\end{bmatrix}\begin{bmatrix}x_p(t) \\ e(t)\end{bmatrix} \le 0 \qquad (21)$$

As long as the event triggering condition is satisfied, the event-triggered control law will keep the actuator signal value constant. This value will be updated whenever the triggering condition is violated.

Corollary [17]: Let $\dot{x}(t) = Ax + Bu$ be a linear control system, let $u = Kx$ be a linear control law rendering the closed loop system globally asymptotically stable. For any initial condition, the inter-event times $t_{i+1} - t_i = \tau$ that implicitly defined by the execution rule $\|e\| = \sigma\|x\|$, are lower bounded by time τ satisfying.

$$\varphi(\tau, 0) = \sigma \qquad (22)$$

Where $\varphi(\tau, \varphi_0) = \sigma$ is the solution of:

$$\dot{\varphi} = |A + BK| + (|A + BK|) + |BK|)\varphi + |BK|\varphi^2 \qquad (23)$$

satisfying $\varphi(0, \varphi_0) = \varphi_0$.

IV. Event-Based Tracking Control of Linear System

Output tracking control (also called model reference control) is one of the most active subjects in control theories. It has wide applications in dynamics process in industry, economics and biology. The main objective of tracking control is to make the output of the plant track the desired trajectory. Here we assume that the plant is completely observable, so designing a controller toward tracking the desired output leads to state trajectory

tracking. Considering x_d, the desired state trajectory, is a smooth signal then we define tracking error as:

$$\varepsilon = x - x_d \qquad (24)$$

Taking derivative from both side of (24) yields:

$$\dot{\varepsilon} = \dot{x} - \dot{x}_d \qquad (25)$$

Substituting plant dynamics

$$\dot{\varepsilon} = Ax + Bu - \dot{x}_d = A\varepsilon + Ax_d + Bu - \dot{x}_d \qquad (26)$$

Taking $w = Ax_d + Bu - \dot{x}_d$, (26) becomes:

$$\dot{\varepsilon} = A\varepsilon + w \qquad (27)$$

This is reduced to the regulator problem we have tackled before. To stabilize the tracking error to zero, we define the pseudo control law: $w = K\varepsilon$ which $A + K$ has negative Eigen values.

V. Optimizing Quadratic Cost Function

So far we have seen that traditional digital control main theory is based on sampling with equidistant intervals. This leads to big loss of energy in wireless devices due to energy it takes to send samples to other devices in network. Using event-based control techniques, this challenge is well confronted and leads to large energy savings. Beside energy issue, network traffic is reduced considerably.

Now we want to go a step further to use optimal controller in an event-based paradigm. A very well tackled problem in classic optimal control theory is Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT). In case of tracking, because the final time is not bounded and the control process takes place in infinite time, minimizing the quadratic cost function (33) doesn't make sense because of ever increasing characteristic of this type of cost functions over time [18]. Thus our focus will be on the infinite LQR problem.

As introduced in optimal control literature, this leads to solving the Algebraic Riccati Equation (ARE) to find optimal control $u = Kx$ to minimize the cost function. This procedure is simply as follows:

For a continuous-time linear system described by

$$\dot{x}(t) = Ax + Bu \qquad (28)$$

With a cost function defined as

$$J = \int_0^\infty (x^TQx + u^TRu + 2x^TNu)dt \qquad (29)$$

The feedback control law that minimizes the value of the cost is

$$u = -Kx \qquad (30)$$

Where K is given by

$$K = R^{-1}(B^TP + N^T) \qquad (31)$$

and P is found by solving the continuous time algebraic Riccati equation.

$$A^TP + PA - (PB + N)R^{-1}(B^TP + N^T) + Q = 0 \qquad (32)$$

In Section VI, simulation results are shown with two examples.

VI. Simulation Results and Hardware Implementation

We now illustrate the previous results on linear control system.

For the first example, consider this second order system with initial condition $x_0 = \begin{bmatrix}0.5 \\ -0.5\end{bmatrix}$.

$$\dot{x} = \begin{bmatrix}-1 & 0 \\ 0 & -2\end{bmatrix}x + \begin{bmatrix}1 \\ 0\end{bmatrix}u \qquad (33)$$

LQR optimal control law obtained as:

$$u = [-1 - 2]x \qquad (34)$$

Now deriving the error signal, we have reached the event trigger condition

$$\|e\| \le 0.8\|x\| \qquad (35)$$

As it can be seen in figure 2, with only 8 samples, the system has reached to the origin. This shows an optimality, in contrast to traditional digital control system, in the network traffic reduction and energy saving in nodes.

In the second example we consider a first order plant with initial condition $x_0 = 1$ and the desired trajectory is a sinusoidal signal as shown below:

$$\dot{x} = -10x + u \tag{36}$$

The desired trajectory is:

$$x_d = 2\sin(2\pi t) \tag{37}$$

The slew rate $\sigma = 0.4$. In figure 4-6 simulation results are shown. Here we added a disturbance source with a uniform random distribution. Its amplitude is 0.2 and can be positive and negative, leading to 10% disturbance. The sampling interval of this disturbance is 0.5 seconds. The simulation time is considered 20 seconds and by only 14 events, the tracking is done effectively. This shows a major improvement in energy saving in compare with traditional digital control system theory that have high frequency sampling rate.

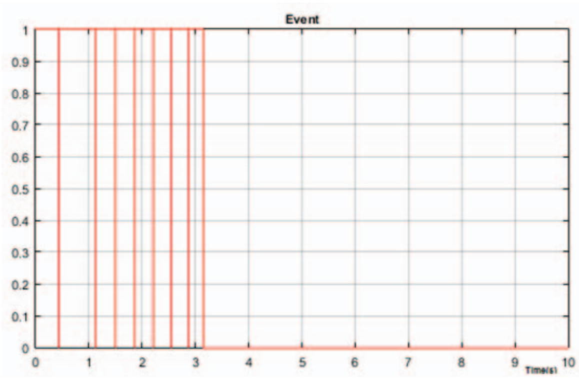

Fig. 1. Example 1 event sequence

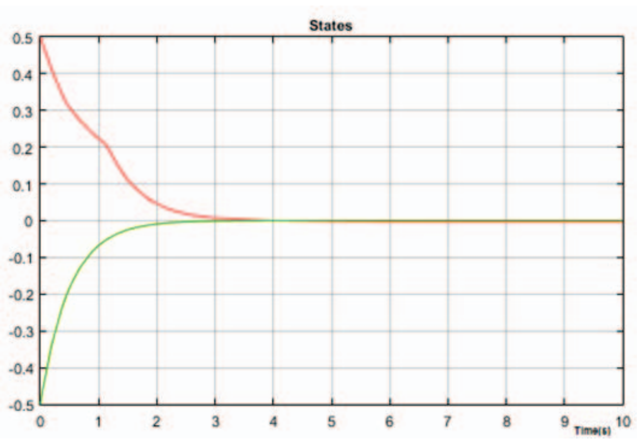

Fig. 2. Example 1 states trajectory

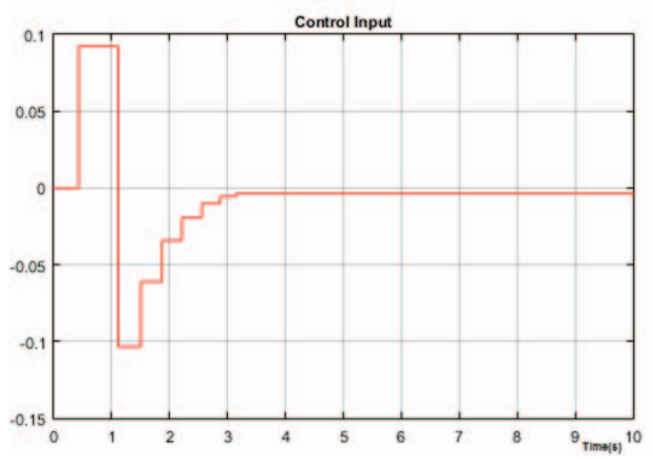

Fig. 3. Example 1 control input

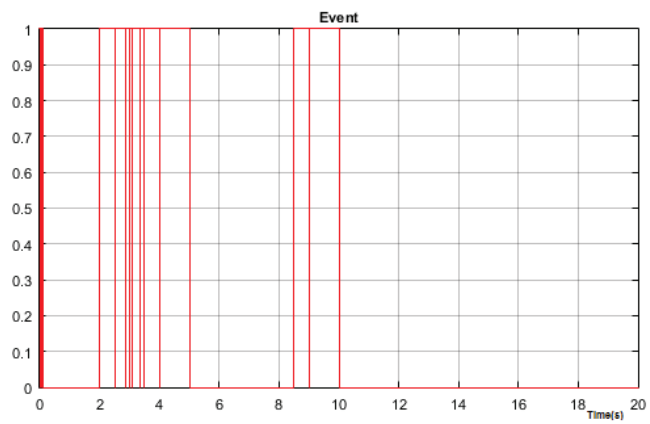

Fig. 4. Example 2 event sequence

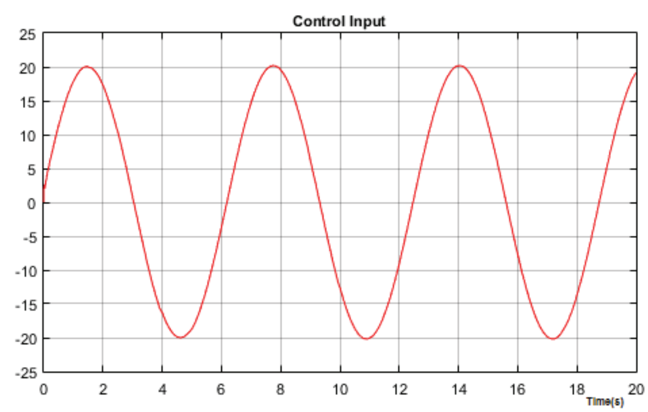

Fig. 5. Example 2 control input

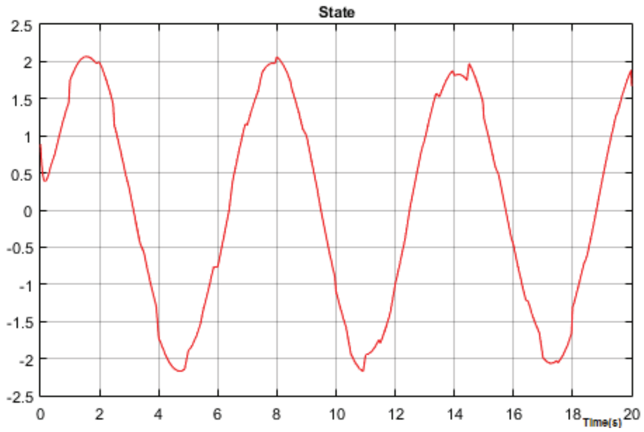

Fig. 6. Example 2 state trajectory

We have implemented both controller and the event trigger parts separately in a hardware device with an 8 bit microcontroller specifically AVR Atmega32 with maximum 16 MHz CPU clock frequency.

The plant simulation is undertaken in SIMULINK. The communication with PC is done via serial interface (RS232). When device is acting as the controller, each transaction on events is quantized as a double value and is sent to the controller via interrupting the CPU. If device is acting as the event trigger, as mentioned in PETC, states are sampled in a periodic fashion (here 1ms) to drive the data needed for evaluating the event triggering condition on device.

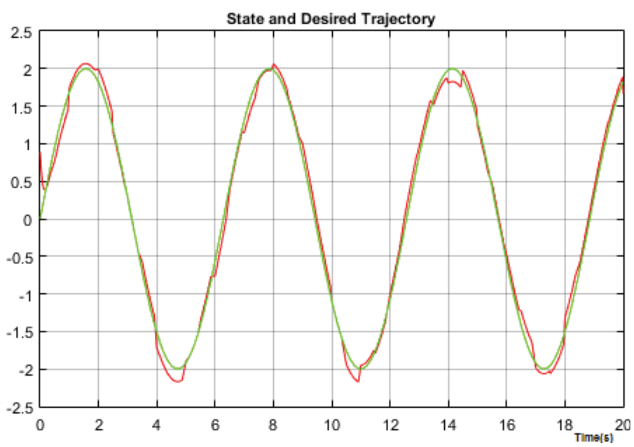

Fig. 7. Example 2 state trajectory and refrence signal

In Figure (8) the architecture of this setup is shown and in Figure (9) the actual setup is demonstrated. The benefit of placing hardware in the control loop is that tests can be done on actual hardware for the developed control algorithm. Results of hardware control are very close to what we have got in simulation.

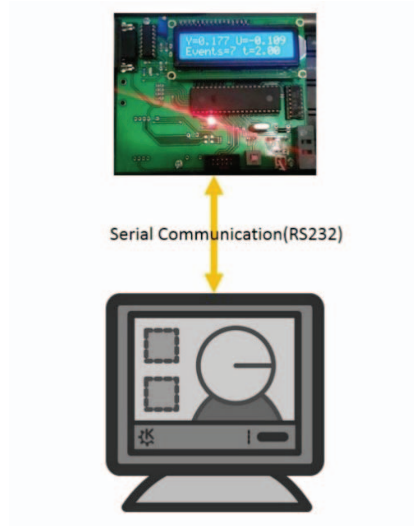

Fig. 8. Test bed communication architecture

VII. CONCLUSION

In this paper, a novel architecture for designing wireless control systems is presented. We demonstrated how to stabilize a nonlinear system with only updating control input at the certain times. We simulate a linear case for ease of presentation and solved the problem for event-based regulation, considering quadratic cost functions. Formulating event-based tracking was done effectively and the results show the effectiveness of our proposed method.

The experimental results show that, by using the proposed method, the energy consumption in battery based nodes and the traffic can be reduced in the network significantly.

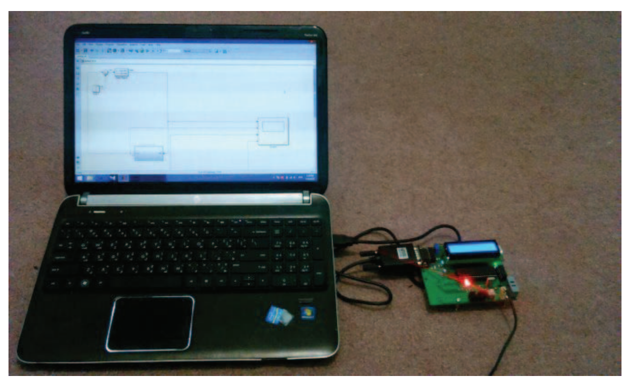

Fig. 9. Test bed hardware in the loop (HIL)

REFERENCES

[1] J. Liu, A. Gusrialdi, D. Obradovic, and S. Hirche, "Study on the effect of time delay on the performance of distributed power grids with networked cooperative control," in Proceedings of the 1st IFAC Workshop on Estimation and Control of Networked Systems, 2009, pp. 168–173.

[2] H. A. Thompson, "Wireless and Internet communications technologies for monitoring and control," Control Engineering Practice, vol. 12, no. 6, pp. 781–791, 2004.

[3] D. Lehmann and J. Lunze, “Extension and experimental evaluation of an event-based state-feedback approach,” Control Engineering Practice, vol. 19, pp. 101–112, 2011.

[4] I. F. Akyildiz and I. H. Kasimoglu, “Wireless sensor and actor networks: Research challenges,” Ad Hoc Networks, vol. 2, no. 4, pp. 351–367, 2004.

[5] K. H. Johansson, M. Törngren, and L. Nielsen, “Vehicle applications of controller area network,” in Handbook of Networked and Embedded Control Systems, D. Hristu-Varsakelis and W. S. Levine, Eds. Boston, MA: Birkhäuser, 2005, ch. 6, pp. 741–765.

[6] K.J. Astrom. Event based control. In A. Astolfi and L. Marconi, editors, Analysis and design of non-linear control systems, pages 127-147. Springer-Verlag, Berlin, 2008

[7] J. Lunze. Event-based control: a tutorial introduction. Journal of Society of Instrument and Control Engineers, 49(11):783-788, 2010.

[8] P. Tabuada, “Event-Triggered Real-Time Scheduling of Stabilizing Control Tasks,” *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[9] A. Eqtami, D. V. Dimarogonas, and K. J. Kyriakopoulos, “Eventtriggered control for discrete-time systems,” Proceedings of the 2010 American Control Conference, pp. 4719–4724, Jun. 2010.

[10] X. Meng and T. Chen, “Event based agreement protocols for multi-agent networks,” Automatica, vol. 49, no. 7, pp. 2125–2132, Jul. 2013.

[11] W. P. M. H. Heemels, M. C. F. Donkers, and A. R. Teel, “Periodic event-triggered control based on state feedback,” IEEE Conference on Decision and Control and European Control Conference, pp. 2571– 2576, Dec. 2011.

[12] W. P. M. H. Heemels and M. C. F. Donkers, “Model-based periodic event-triggered control for linear systems,” Automatica, vol. 49, no. 3, pp. 698–711, Mar. 2013.

[13] K.-E. Årzén, “A simple event-based PID controller,” in Preprints 14th World Congr. IFAC, Beijing, China, Jan. 1999.

[14] K. Åström and B. Bernhardsson, “Systems with Lebesgue sampling,” in Directions in Mathematical Systems Theory and Optimization, A. Rantzer and C. Byrnes, Eds. Berlin/Heidelberg, Germany: Springer, 2003, vol. 286, Lecture Notes in Control and Information Sciences, pp. 1–13.

[15] J. Sandee, “Event-Driven Control in Theory and Practice,” Ph.D. dissertation, Technische Universiteit Eindhoven, Eindhoven, The Netherlands, 2006.

[16] X. Wang and M. Lemmon, “Self-triggering under state-independent disturbances,” IEEE Trans. Autom. Control, vol. 55, no. 6, pp. 1494–1500, Jun. 2010.

[17] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, “An introduction to event-triggered and self-triggered control,” in Proc. IEEE Conf. Decision Control, 2012, pp. 3270–3285.

[18] R. Alba-Flores ,E. Barbieri, "Real-time Infinite Horizon Linear-Quadratic Tracking Controller for Vibration Quenching in Flexible Beams" IEEE International Conference on Systems, Man and Cybernetics, 2006, pp.38-43 Oct. 2006.